An Adaptive Derivative-free Nonlinear Kalman Filtering Approach using Flat Inputs ⋆

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Abstract:
The concept of flat inputs has been proved to be a valuable tool for control design if the system is non-differentially flat. Recently, flat inputs have also found to be useful to provide efficient solutions to solve state estimation problems for these systems. However, to the best of our knowledge no studies have yet investigated the impact of incorrectly describing the noise statistics in the accuracy of state estimation for Kalman filter approaches using flat inputs. Considering such background, this paper introduces an adaptive state estimation-based control strategy in order to handle the unknown process noise covariance for observable non-differentially flat nonlinear systems as long as the internal dynamics of the system are stable. A numerical simulation of an underactuated ship was performed to demonstrate the effectiveness of the proposed adaptive method.

Keywords:
Non-differentially flat nonlinear systems; Flat inputs; Nonlinear Control; Adaptive Kalman filter; Unknown process noise covariance; Underactuated ship

1. INTRODUCTION

Numerous studies in the literature routinely make use of Kalman filter theory, which is known for its wide application in state estimation problems. It is well known that the optimality of the Kalman filter relies on the assumptions that (i) the process and measurement noises are zero-mean white Gaussian noises with known covariance matrices and (ii) a linear dynamic system model has been precisely obtained *a priori* (Kalman, 1960).

Unfortunately, these assumptions are unlikely to hold in practice and, consequently, may affect the accuracy of state estimation of the standard Kalman filter. For example, the incorrect description of the noise statistics may degrade the control performance even destabilize the system. This explains why there has been a growing focus in the last few decades on the development of methods for noise covariance matrices estimation (see Mehra (1972); Li and Bar-Shalom (1994); Duník et al. (2009, 2017); Kost et al. (2022) and references therein).

In parallel, some effort has also been made to deal with cases that violate the assumption of the Kalman filter regarding system linearity in the literature. Traditional estimation techniques for nonlinear systems such as the extended Kalman filter (EKF) (Schmidt, 1981) and the unscented Kalman filter (UKF) (Julier and Uhlmann, 1997) are known to be a good approximation of an optimal estimate in a variety of control applications for nonlinear systems. A more efficient implementation of nonlinear Kalman filtering was introduced in the work of Rigatos (2012) and widely applied to state estimation-based control of a special class of nonlinear systems (also known as differentially flat systems). Known as the derivative-free Kalman filter (DKF), the DKF performs a nonlinear state estimation without the need for derivatives and Jacobians calculation unlike EKF and provides an estimation accuracy equivalent to that of the UKF, but with less computational cost mainly because it follows the fast recursion of the standard Kalman filter.

Although numerous practical examples have been shown to belong to the class of differentially flat systems in the literature (see Fliess et al. (1995)), some classic nonlinear systems (e.g., inverted double pendulum, marine vessels) do not belong to it, which restricts the application of the aforementioned DKF approach to state estimation-based control. Therefore, increasing attention has been placed on the development of control strategies for non-differentially flat nonlinear systems.

Within this context, in our recent work (Limaverde Filho et al., 2021), we have derived a state estimation-based control strategy for observable non-differentially flat nonlinear systems with stable internal dynamics by combining the tracking control method based on the concept of flat inputs described in Stumper et al. (2009) with the DKF approach proposed in the work of Rigatos (2012). A major drawback of our method, however, is that we assume a prior knowledge about the noise statistics affecting the states and measurements, which is not necessarily true for most practical applications.

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To address this issue, the primary contribution of this work is to incorporate the adaptive DKF approach presented in Limaverde Filho et al. (2022) that estimates in real time the process noise covariance matrix by considering the impact of the control signal on the variance of state variables into the tracking control method based on flat inputs described in Stumper et al. (2009). As result, it is expected to obtain a better performance of the nonlinear controllers in regarding the approach presented in Limaverde Filho et al. (2021).

2. DIFFERENTIAL FLATNESS THEORY

Differential flatness theory has been introduced in control theory by Fliess et al. (1992) and its principles have been widely used for both tracking control and state estimation of nonlinear systems (Rigatos, 2015). In this section, we recall some elements related to differential flatness theory that will be used in the later sections.

Consider the input-affine nonlinear MIMO system in state space model of the form:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u = f(x) + \sum_{i=1}^{m} g_i(x)u_i \\ y_i &= h_i(x), \qquad i = 1, \cdots, m \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u_i,\ y_i \in \mathbb{R}$, f and $g_1,\ \cdots, g_m$ are smooth vector fields, and h_i are smooth functions.

Roughly speaking, the nonlinear system (1) is called differentially flat if there exists a vector $y_z \in \mathbb{R}^m$ as below:

$$y_z = \phi_z(x, u, \dot{u}, \cdots, u^{(l)}), \tag{2}$$

with the state and input expressions parameterized as follows:

$$x = \phi_x(y_z, \dot{y}_z, \cdots, y_z^{(q)}) \tag{3}$$

$$u = \phi_u(y_z, \dot{y}_z, \cdots, y_z^{(q)}, y_z^{(q+1)}) \tag{4}$$

where ϕ_z, ϕ_x and ϕ_u are smooth vector functions. The vector y_z represents the flat outputs of the system with l and q being finite numbers. In the particular case that y_z are exclusively functions of the state vector, we say that the system is x-flat.

If the system (1) is differentially flat and admits a static feedback linearization (i.e., a change of coordinates for both the system state variables and the system's control input), then (1) is x-flat and can be transformed in the Brunovsky canonical form:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \vdots \\ \dot{z}_{r_1-1} = z_{r_1} \\ \dot{z}_{r_1} = F_1(z) + \sum_{j=1}^{m} G_{1,j}(z)u_j \\ \dot{z}_{r_1+1} = z_{r_1+2} \\ \dot{z}_{r_1+2} = z_{r_1+3} \\ \vdots \\ \dot{z}_{n-1} = z_n \\ \dot{z}_n = F_m(z) + \sum_{j=1}^{m} G_{m,j}(z)u_j \end{cases} \qquad \begin{cases} y_{z_1} = z_1 \\ y_{z_2} = z_{r_1+1} \\ \vdots \\ y_{z_m} = z_{n-r_m+1} \end{cases} \tag{5}$$

where $z = [z_1\ \cdots\ z_n]^T$ is the new state vector, $y_z = [y_{z_1}\ \cdots\ y_{z_m}]^T$ is new the output vector (and also the flat output vector of (1)), F_i are the drift functions and $G_{i,j},\ i,j =\ 1,2,\cdots,m$ are smooth functions corresponding to the control input gains. Also, it holds that $r_1 + r_2 + \cdots + r_m = n$. To transform (1) into the canonical form described in (5) is called static feedback linearization problem (see Jakubczyk and Respondek (1980); Hunt and Su (1981); Isidori (1995) for more details).

Remark 1. Although it is also well known in the literature that there exist differentially flat nonlinear systems that can be written in Brunovsky canonical form through a dynamic feedback linearization (Fliess et al., 1995), our discussion in this paper focuses on differentially flat nonlinear systems that are linearizable by static state feedback.

System (5) can be expressed in the following input-output representation:

$$y_{z_i}^{(r_i)} = F_i(z) + \sum_{j=1}^{m} G_{i,j}(z)u_j, \qquad i = 1, \cdots, m \tag{6}$$

which can also be written in the state space form as:

$$\begin{cases} \dot{z} = A_b z + B_b \upsilon \\ y_z = C_b z \end{cases} \tag{7}$$

with $A_b = diag\{A_1,\ \cdots,\ A_m\}$, $B_b = diag\{B_1,\ \cdots,\ B_m\}$ and $C_b = diag\{C_1,\ \cdots,\ C_m\}$, whose elements are defined as:

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{r_i \times r_i}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{1 \times r_i}, \quad C_i^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{1 \times r_i} \tag{8}$$

and the new control input υ is defined as:

$$\upsilon = F(z) + G(z)u, \tag{9}$$

where $F(z) = [F_1(z)\ \cdots\ F_m(z)]^T$ and $G(z) = [G_1(z)\ \cdots\ G_m(z)]^T$ with $G_i(z) = [G_{1,i}(z)\ \cdots\ G_{m,i}(z)]^T$.

Provided that all state variables of (1) are measured, then a state-feedback control law can be formulated as:

$$u = G^{-1}(z)\left[\upsilon - F(z)\right], \tag{10}$$

where υ can be designed based on linear control theory for trajectory tracking purposes.

For many practical uses, unfortunately, it may not always be cost-effective to measure all the states required to control the system. In this context appeared the derivative-free nonlinear Kalman filtering approach proposed in the work of Rigatos (2012) for differentially flat nonlinear systems. More precisely, for (7), it is straightforward to apply the standard Kalman filter recursion after carrying out discretization of matrices A_b, B_b and C_b with common discretization methods. Therefore, the estimated state vector of (7) can be used to compute the nonlinear control law according to (10).

3. FLAT INPUTS-BASED STATE FEEDBACK ADAPTIVE CONTROL DESIGN

As a dual perspective of flat outputs, the concept of flat inputs has been introduced as a problem of actuator placement (Waldherr and Zeitz, 2008, 2010). To illustrate this statement, let us consider the following observable nonlinear MIMO system in state space model of the form:

$$\begin{aligned} \dot{x} &= f(x,u) \\ y_i &= h_i(x), \qquad i = 1, \cdots, m \end{aligned} \tag{11}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y_i \in \mathbb{R}$, f is a smooth vector field, and h_i are smooth functions.

If there exists m input vector fields $\gamma_i(x)$ such that the resulting input-affine nonlinear MIMO system $\dot{x} = f(x,0) + \sum_{i=1}^{m} \gamma_i(x) u_{i_f}$ is x-flat with y_i being a flat output, then the corresponding u_{i_f} is called a flat input.

Remark 2. The solutions of the resulting input-affine nonlinear MIMO system, here called system with flat inputs, may differ from those of (11), despite their output vectors being identical.

For system (11) without an external input ($u \equiv 0$), here called observed uncontrolled system, the notion of observability quasi-indices (see Krener and Respondek (1985); Bingulac and Krtolica (1987)) can be used to compute the input vector fields associated with the flat inputs, as described in Waldherr and Zeitz (2010); Nicolau et al. (2020).

If the observed uncontrolled system has observability quasi-indices $\kappa^O = (\kappa_1^O, \cdots, \kappa_m^O)$, the matrix

$$E_O = \frac{\partial}{\partial x} \begin{bmatrix} h_1(x) \\ L_f h_1(x) \\ \vdots \\ L_f^{\kappa_1^O - 1} h_1(x) \\ \vdots \\ h_m(x) \\ L_f h_m(x) \\ \vdots \\ L_f^{\kappa_m^O - 1} h_m(x) \end{bmatrix} \tag{12}$$

has rank n and $\sum_{i=1}^{m} \kappa_i^O = n$ with $\kappa_i^O \geq 0$, $i = 1, \cdots, m$. Then, the input vector fields can be computed as follows:

$$\gamma_M = [\gamma_1 \ \cdots \ \gamma_m] = E_O^{-1} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_m \end{bmatrix}^T A_{FIM} \tag{13}$$

where $A_{FIM} \in \mathbb{R}^{m \times m}$ is a non-singular matrix composed of arbitrary scalar functions of the state vector, while ϕ_j are n-dimensional row vectors of the form:

$$\phi_j = [0, \cdots, 0, 1, 0, \cdots, 0], \qquad j = 1, \cdots, m \tag{14}$$

with the 1 in the $\left(\sum_{i=1}^{j} \kappa_i^O\right)$-th position.

In the MIMO case, it has also been shown that the observability is not necessary for the existence of flat inputs, and algorithms to compute the input vector fields associated with the flat inputs for non-observable systems were recently reported in the related literature (Nicolau et al., 2018, 2020). However, we will limit our analysis and discussion here to the observable case.

As stated in Waldherr and Zeitz (2010), a flat input must be realized as a physical actuator. However, this may not be possible for any given system due to design constraints, even if in principle a flat input exists. In Stumper et al. (2009), as long as the internal dynamics of the system are stable, the authors proposed to overcome this issue by adding a dynamic compensator $\left(u_f \to u\right)$ in the form of a prefilter, which is designed such that the input-output behavior of the extended system (i.e., series connection of the dynamic compensator with the original system) is equal to that of the system with flat inputs. In order to force the input-output behavior for both systems to be identical, we first obtain the κ_i^O-th time derivative of y_i, $i = 1, \cdots, m$, for the original system in such a way that it is expressed only in terms of y, u and their respective time derivatives. The same procedure is then also done for the system with flat inputs, but the expressions obtained are in terms of y, u_f and their respective time derivatives. From these expressions, it is possible, therefore, to construct a system of differential equations whose solution for the real control input u corresponds to the desired dynamic compensator. Accordingly, it is sufficient to design the conventional flatness-based controller described in Section 2 for the system with flat inputs.

As first presented in Limaverde Filho et al. (2021), the findings reported in Stumper et al. (2009) offered a new way to handle state estimation for observable non-differentially flat nonlinear systems with stable internal dynamics. More precisely, since the output vector y of the original system is identical to that of the system with flat inputs, the DKF can be applied to perform state estimation in the system with flat inputs using the measurements from the original system. Then, the expression of the controller u_f is obtained directly from the flatness property of the system with flat inputs by using the estimates of the flat outputs and their respective time derivatives. The control input u is finally computed through the dynamic compensator. On the other hand, how process and measurement noise statistics of the nonlinear system are related to those of the system with flat inputs has not been addressed by the authors.

It is straightforward to observe that the measurement noise covariance remains the same for the system with flat inputs, since its output vector is identical to that of the original system. Regarding the process noise, it is not a easy task to obtain an analytical expression between the covariance matrices. For example, even with the assumption of a constant process noise covariance matrix for the original system, it may be not valid for the system with flat inputs written in Brunovsky canonical form, further corroborating the need to incorporate adaptive methodologies to the DKF-based state estimation approach using flat inputs proposed in Limaverde Filho et al. (2021).

To address this issue, in this paper we proposed to combine the tracking control method based on flat inputs described in Stumper et al. (2009) with the adaptive DKF approach presented in Limaverde Filho et al. (2022), as can be seen in Figure 1. Through the closed-loop joint analysis of the a priori recursive form of the Kalman filter and the system with flat inputs written in Brunovsky canonical form, we adaptively estimate in real-time its process noise covariance by relating it to the observation vector covariance. The latter is obtained from an exponential moving average technique. As result, it is expected to improve the performance of the nonlinear controllers in regarding the approach outlined in Limaverde Filho et al. (2021).

In what follows, we first apply the proposed adaptive state estimation-based control scheme using flat inputs for an underactuated ship. Then, a numerical example is provided to illustrate the effectiveness of the proposed adaptive approach.

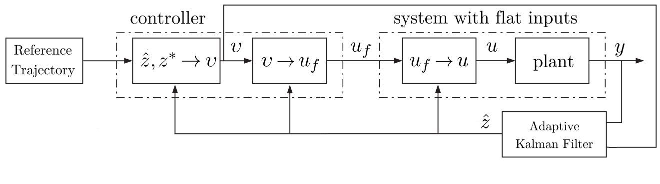

Figure 1. The proposed adaptive state estimation-based control scheme using flat inputs.

4. UNDERACTUATED SHIP

The underactuated ship is an example of a non-differentially flat nonlinear system and can be described by the following mathematical model (Sira-Ramírez, 1999):

$$\begin{cases} \dot{x} & = u\cos(\psi) - v\sin(\psi) \\ \dot{y} & = u\sin(\psi) + v\cos(\psi) \\ \dot{\psi} & = r \\ \dot{v} & = -\eta u r - \beta v \end{cases} \tag{15}$$

where x, y and ψ determine the position and orientation of the ship in the Earth-fixed frame, which are the only measurable state variables of the system. The state variable v represents the sway velocity in the Body-fixed reference frame. The constants η and β are strictly positive constants that depend on the structural characteristics of the systems. The control inputs variables u and r represent the surge and yaw velocities, respectively, in the Body-fixed reference frame.

As stated in Consolini and Tosques (2012), if the output of (15) is the center of mass $(x,\ y)$, the system is non-minimum phase with respect to this output. Then, the non-observable internal dynamics is unstable, which would not allow us to apply the proposed adaptive state estimation-based control scheme using flat inputs.

For this reason, following the idea of output redefinition suggested in Consolini and Tosques (2012), we define:

$$\Omega = \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix} = \begin{bmatrix} x + \rho\cos(\psi) \\ y + \rho\sin(\psi) \end{bmatrix}, \qquad \rho > 0 \tag{16}$$

and choose Ω as the system output. This point is on the ship symmetry axis, ahead of the ship at a distance ρ from the center of mass (see Figure 2).

Roughly speaking, Consolini and Tosques (2012) proved that, given smooth desired trajectory for the redefined output vector Ω and if ρ is sufficiently large, the non-observable internal dynamics can be made arbitrarily small provided that the variations of the time derivatives of the specified trajectory up to the third order are sufficiently small. Throughout this paper, these conditions are, therefore, assumed to be valid in order to transform a non-minimum phase problem into a problem with a stable internal dynamics.

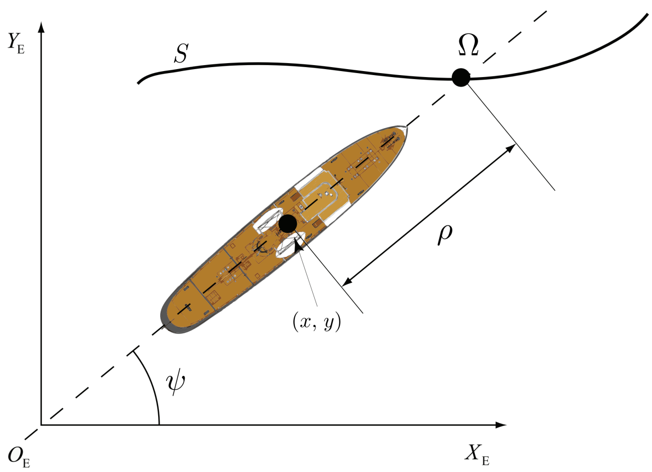

Figure 2. The underactuated ship with the redefined output vector Ω. Adapted from Consolini and Tosques (2012).

Let the observed uncontrolled system of (15) be given by:

$$\begin{cases} \dot{x} & = -v\sin(\psi) \\ \dot{y} & = v\cos(\psi) \\ \dot{\psi} & = 0 \\ \dot{v} & = -\beta v \end{cases} \tag{17}$$

where the system outputs are $\Omega_1 = x + \rho\cos(\psi)$ and $\Omega_2 = y + \rho\sin(\psi)$.

As presented in Limaverde Filho et al. (2021), when choosing the tuple $\kappa^O = (2,2)$ as the observability indices for (17), the system with flat inputs can be described by:

$$\begin{cases} \dot{x}_{1_f} & = \dfrac{u_{1_f}\rho\sin(2x_{3_f})}{2x_{4_f}} + \dfrac{u_{2_f}\rho\left(\sin(x_{3_f})\right)^2}{x_{4_f}} - x_{4_f}\sin(x_{3_f}) \\ \dot{x}_{2_f} & = x_{4_f}\cos(x_{3_f}) - \dfrac{u_{1_f}\rho\left(\cos(x_{3_f})\right)^2}{x_{4_f}} - \dfrac{u_{2_f}\rho\sin(2x_{3_f})}{2x_{4_f}} \\ \dot{x}_{3_f} & = \dfrac{u_{1_f}\cos(x_{3_f})}{x_{4_f}} + \dfrac{u_{2_f}\sin(x_{3_f})}{x_{4_f}} \\ \dot{x}_{4_f} & = u_{1_f}\sin(x_{3_f}) - u_{2_f}\cos(x_{3_f}) - \beta x_{4_f} \end{cases} \tag{18}$$

where $x_f = \left[x_{1_f}\ x_{2_f}\ x_{3_f}\ x_{4_f}\right]^T$ represents the solution of (18), which is not necessarily identical to the solution of (15). In addition, $y_z = [y_{z_1}\ y_{z_2}]^T = \left[x_{1_f} + \rho\cos(x_{3_f})\ x_{2_f} + \rho\sin(x_{3_f})\right]^T$ corresponds to both output vector and flat output vector of (18), which are equal to the redefined output vector of (15).

The differential parametrization of the states and inputs of (18) in terms of the flat outputs and their time derivatives can also be founded in Limaverde Filho et al. (2021). Accordingly to the flatness property of (18), the system can be transformed into a linear system in the Brunovsky canonical form as follows [1]:

$$\begin{cases} \ddot{y}_{z_1} & = F_1(z_f) + G_{1,1}(z_f)u_{1_f} = \upsilon_1 \\ \ddot{y}_{z_2} & = F_2(z_f) + G_{2,2}(z_f)u_{2_f} = \upsilon_2 \end{cases} \tag{19}$$

where $z_f = [y_{z_1}\ \dot{y}_{z_1}\ y_{z_2}\ \dot{y}_{z_2}]^T$ and the pair (υ_1, υ_2) representing the new control variables.

This implies that (19) can be written in the following state-space representation:

$$\begin{cases} \dot{z}_f & = A_b z_f + B_b \upsilon \\ y_z & = C_b z_f \end{cases} \tag{20}$$

with

$$A_b = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ B_b = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},\ C_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},\ \upsilon = \begin{bmatrix} \upsilon_1 \\ \upsilon_2 \end{bmatrix} \tag{21}$$

By assuming now that the measurement noise covariance matrix for aforementioned model is known *a priori*, we simultaneous estimate its process noise covariance matrix and state variables using the adaptive DKF approach presented in Limaverde Filho et al. (2022). The estimates of state vector, $\hat{z}_f$, can then be used to design the controllers υ_1 and υ_2 for (20) as follows:

[1] The expressions of $F_1(z_f)$, $F_2(z_f)$, $G_{1,1}(z_f)$ and $G_{2,2}(z_f)$ were omitted to facilitate the reader's understanding.

$$\upsilon_1 = \ddot{y}^*_{z_1} - k_{11}\left[\hat{\dot{y}}_{z_1} - \dot{y}^*_{z_1}\right] - k_{01}\left[\hat{y}_{z_1} - y^*_{z_1}\right] \quad (22)$$

$$\upsilon_2 = \ddot{y}^*_{z_2} - k_{12}\left[\hat{\dot{y}}_{z_2} - \dot{y}^*_{z_2}\right] - k_{02}\left[\hat{y}_{z_2} - y^*_{z_2}\right] \quad (23)$$

where the desired trajectories $y^*_{z_1}$, $\dot{y}^*_{z_1}$, $\ddot{y}^*_{z_1}$, $y^*_{z_2}$, $\dot{y}^*_{z_2}$ and $\ddot{y}^*_{z_2}$ are assumed to be known and the controller gains (k_{11}, k_{01}) and (k_{12}, k_{02}) are chosen such that $p_1(s) = s^2 + k_{11}s + k_{01}$ and $p_2(s) = s^2 + k_{12}s + k_{02}$ are Hurwitz polynomials.

From (19), note that the expressions of the controllers u_{1_f} and u_{2_f} are given by:

$$u_{1_f} = G^{-1}_{1,1}(\hat{z}_f)\left[\upsilon_1 - F_1(\hat{z}_f)\right] \quad (24)$$

$$u_{2_f} = G^{-1}_{2,2}(\hat{z}_f)\left[\upsilon_2 - F_2(\hat{z}_f)\right] \quad (25)$$

Finally, we compute the dynamic compensator by requiring the input-output behavior of (15) and (18) to be equal. This results in a system of differential equations whose solutions correspond to the following dynamic compensator:

$$\dot{u} = \lambda_u(\hat{z}_f, u_{1_f}, u_{2_f}, u, r) \quad (26)$$

$$\dot{r} = \lambda_r(\hat{z}_f, u_{1_f}, u_{2_f}, u, r) \quad (27)$$

where λ_u and λ_r are smooth functions. The control signals u and r of the underactuated ship are then obtained by integrating (26) and (27).

5. NUMERICAL SIMULATION

To illustrate the effectiveness of the proposed adaptive state estimation-based control scheme using flat inputs, a numerical study was conducted for an underactuated ship of form (15) with parameters $\eta = 0.6745$ and $\beta = 0.6577$ extracted from Pan and Do (2009). The ship length is assumed to be 32 m and the center of mass situated in the center of the ship. Then, we set $\rho = 16$ m so that the reference point Ω defined in (16) corresponds to the ship bow.

The initial condition of the ship is selected to be $x(0) = -10\ m$, $y(0) = 0\ m$, $\psi(0) = \pi/3\ rad$ and $v(0) = 0\ m/s$. By setting the simulation duration to 100 s with a sampling time of 0.1 s, the desired trajectory for the position of the ship in the Earth-fixed frame is defined as follows:

$$x^*(t) = 50\sin(0.1t), \qquad y^*(t) = 50\cos(0.1t) \quad (28)$$

where ψ^*, v^*, u^* and r^* can be calculated using the formulas presented in Sira-Ramírez (1999). As a result, the desired trajectory for the redefined outputs and their time derivatives can also be determined. It is also noted that the controller gains were chosen from experience to be the coefficients of $p_1(s) = (s+1)^2$ and $p_2(s) = (s+0.75)^2$, respectively.

The comparison of the results obtained using the proposed adaptive derivative-free nonlinear Kalman Filter based on flat inputs (ADKF-FI) with the Limaverde Filho et al. (2022) method (DKF-FI) is discussed. For both cases, assuming the measurement noise covariance matrix is known *a priori* and equal to $R = 10^{-2}I_{2\times 2}$ throughout the simulation, the process noise covariance matrix was initialized with values $Q = 10^2 I_{4\times 4}$. In addition, the initial estimate of the state vector and its error covariance matrix were set to zero and $I_{4\times 4}$, respectively. The simulation results are illustrated in Figures 3-5.

From Figure 3, we observe that for both methods the underactuated ship in the XY-plane converged to the desired trajectory specified previously. This is corroborated by Figure 4, where we see that all system states towards the desired trajectories in finite time. Besides that, it is clear from Figure 5 that as the ADKF-FI estimates the process noise covariance matrix in the Brunovsky domain, the control signals tend to be smoother than those of the DKF-FI, whose process noise covariance matrix is constant throughout the simulation.

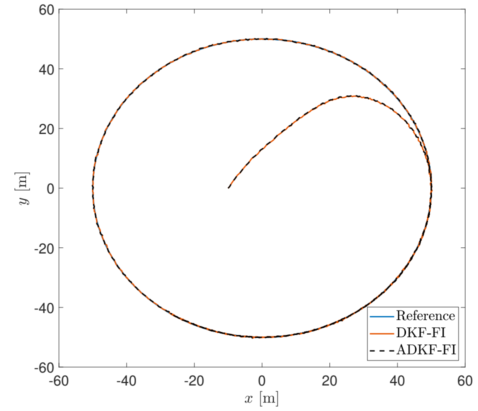

Figure 3. Circular trajectory tracking of the underactuated ship.

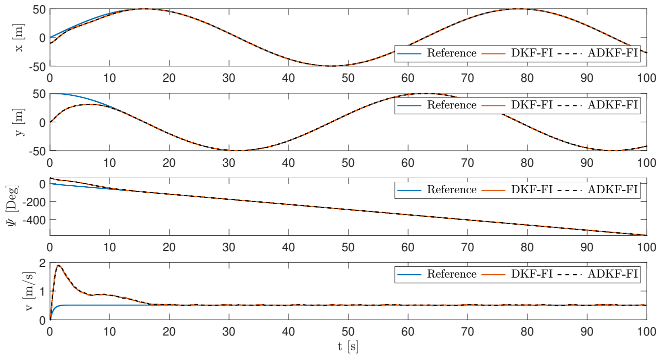

Figure 4. Time evolution of the nonlinear system state variables.

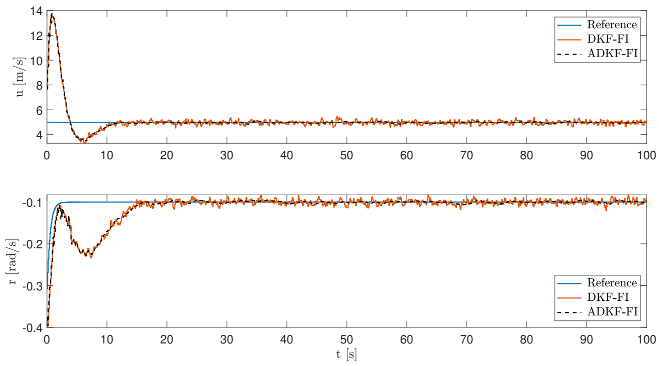

Figure 5. Time evolution of the surge and yaw velocities.

Lastly, to quantitatively evaluate the results, some traditional control performance indices and estimation accuracy metrics were calculated for each method, as shown in Table 1. In general, the findings are in agreement with the conclusions stated by the analysis of the figures above. From the control performance indexes, it can be noted that the use of the adaptive Kalman filter led to a reduction of approximately 75% in the total variation of the control signal without significant changes in the closed-loop output response, representing a notable reduction of the actuator wear. From the estimation accuracy metrics, the RMSE and MAE of the unmeasured states obtained by the ADKF-FI are smaller than that by the DKF-FI.

Table 1. Control performance indexes and state estimation accuracy metrics.

Index/Metric	Variable	DKF-FI	ADKF-FI
ISE	x	2.390×10^2	2.401×10^2
	y	5.186×10^3	5.160×10^3
IAE	x	5.537×10^1	5.570×10^1
	y	1.908×10^2	1.903×10^2
ITAE	x	6.316×10^2	6.439×10^2
	y	9.572×10^2	9.740×10^2
TV	u	1.097×10^1	2.530×10^0
	r	4.224×10^{-1}	1.157×10^{-1}
RMSE	$\dot{y}_{z_1}$	6.791×10^{-1}	4.951×10^{-1}
	$\dot{y}_{z_2}$	7.774×10^{-1}	6.190×10^{-1}
MAE	$\dot{y}_{z_1}$	5.382×10^{-1}	1.047×10^{-1}
	$\dot{y}_{z_2}$	6.195×10^{-1}	1.417×10^{-1}

6. CONCLUSION

In this study, we investigated how inaccurately describing noise statistics affects the performance of a derivative-free Kalman filter-based control method using the concept of flat inputs for observable non-differentially flat nonlinear systems with stable internal dynamics. The findings showed improved controller performance when estimating the process noise covariance matrix associated with the system with flat inputs written in Brunovsky canonical form. Future works will be concerned with taking into account the presence of unknown disturbances. In addition, we expect to extend our research to be applied to unobservable or weakly observable systems such oil reservoirs.

REFERENCES

Bingulac, S. and Krtolica, R. (1987). On admissibility of pseudoobservability and pseudocontrollability indexes. *IEEE Transactions on Automatic Control*, 32(10), 920–922.

Consolini, L. and Tosques, M. (2012). A minimum phase output in the exact tracking problem for the nonminimum phase underactuated surface ship. *IEEE Transactions on Automatic Control*, 57(12), 3174–3180.

Duník, J., Šimandl, M., and Straka, O. (2009). Methods for estimating state and measurement noise covariance matrices: Aspects and comparison. *IFAC Proceedings Volumes*, 42(10), 372–377. 15th IFAC Symposium on System Identification.

Duník, J., Straka, O., Kost, O., and Havlík, J. (2017). Noise covariance matrices in state-space models: A survey and comparison of estimation methods - Part I. *International Journal of Adaptive Control and Signal Processing*, 31(11), 1505–1543.

Fliess, M., Lévine, J., Martin, P., and Rouchon, P. (1992). Sur les systèmes non linéaires différentiellement plats. *Comptes rendus de l'Académie des Sciences*, 315, 619–624.

Fliess, M., Lévine, J., Martin, P., and Rouchon, P. (1995). Flatness and defect of nonlinear systems: Introductory theory and examples. *International Journal of Control*, 61(6), 1327–1361.

Hunt, L. and Su, R. (1981). Linear equivalents of nonlinear time varying systems. In *Proceedings of the MTNS*, 119–123. Santa Monica, CA.

Isidori, A. (1995). *Nonlinear Control Systems*. Springer-Verlag, London.

Jakubczyk, B. and Respondek, W. (1980). On linearization of control systems. *Bulletin de l'Academie Polonaise des Sciences, Series des Sciences Mathematiques*, 28, 517–522.

Julier, S.J. and Uhlmann, J.K. (1997). A new extension of the Kalman filter to nonlinear systems. In *Proceedings of the 11th International Symposium on Aerospace/Defence Sensing, Simulation and Control*, 182–193. SPIE.

Kalman, R.E. (1960). A new approach to linear filtering and prediction problems. *Journal of Basic Engineering*, 82(1), 35–45.

Kost, O., Duník, J., and Straka, O. (2022). Measurement difference method: A universal tool for noise identification. *IEEE Transactions on Automatic Control*, 68(3), 1792–1799.

Krener, A.J. and Respondek, W. (1985). Nonlinear observers with linearizable error dynamics. *SIAM Journal on Control and Optimization*, 23(2), 197–216.

Li, X.R. and Bar-Shalom, Y. (1994). A recursive multiple model approach to noise identification. *IEEE Transactions on Aerospace and Electronic Systems*, 30(3), 671–684.

Limaverde Filho, J.O.A., Fortaleza, E.L.F., and de Campos, M.C.M.M. (2021). A derivative-free nonlinear Kalman filtering approach using flat inputs. *International Journal of Control*, 0(0), 1–11.

Limaverde Filho, J.O.A., Fortaleza, E.L.F., Silva, J.G., and de Campos, M.C.M.M. (2022). Adaptive Kalman filtering for closed-loop systems based on the observation vector covariance. *International Journal of Control*, 95(7), 1731–1746.

Mehra, R. (1972). Approaches to adaptive filtering. *IEEE Transactions on Automatic Control*, 17(5), 693–698.

Nicolau, F., Respondek, W., and Barbot, J.P. (2018). Constructing flat inputs for two-output systems. In *Proceedings of the 23rd International Symposium on Mathematical Theory of Networks and Systems*, 414–421. Hong Kong.

Nicolau, F., Respondek, W., and Barbot, J.P. (2020). Flat inputs: Theory and applications. *SIAM Journal on Control and Optimization*, 58(6), 3293–3321.

Pan, J. and Do, K.D. (2009). *Control of Ships and Underwater Vehicles: Design for Underactuated and Nonlinear Marine Systems*. Springer, London, UK.

Rigatos, G.G. (2012). A derivative-free Kalman filtering approach to state estimation-based control of nonlinear systems. *IEEE Transactions on Industrial Electronics*, 59(10), 3987–3997.

Rigatos, G.G. (2015). *Nonlinear Control and Filtering Using Differential Flatness Approaches: Applications to Electromechanical Systems*. Studies in Systems, Decision and Control (SSDC). Springer.

Schmidt, S.F. (1981). The Kalman filter - Its recognition and development for aerospace applications. *Journal of Guidance and Control*, 4(1), 4–7.

Sira-Ramírez, H. (1999). On the control of the underactuated ship: A trajectory planning approach. In *Proceedings of the 38th IEEE Conference on Decision and Control*, 2192–2197. Phoenix, USA.

Stumper, J.F., Svaricek, F., and Kennel, R. (2009). Trajectory tracking control with flat inputs and a dynamic compensator. In *Proceedings of the 2009 European Control Conference (ECC)*, 248–253. Budapest, Hungary.

Waldherr, S. and Zeitz, M. (2008). Conditions for the existence of a flat input. *International Journal of Control*, 81(3), 439–443.

Waldherr, S. and Zeitz, M. (2010). Flat inputs in the MIMO case. In *Proceedings of the 8th IFAC Symposium on Nonlinear Control Systems*, 695–700. Bologna, Italy.