Synchronization on Van der Pol Oscillators and Application in Micro-Thermal-Fluid Cooling Systems ⋆

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Abstract: In this work, a closed-loop control for stabilizing a set of identical oscillatory systems, interconnected in parallel and with only one common input for all, is discussed. Our focus is primarily on oscillators resembling the Van der Pol type. To achieve this goal, a control strategy for synchronizing the oscillators is evaluated, using their phase response curve (PRC), thereby aligning them under identical conditions. This control law is applied to a flow micro-thermal-fluid cooling system, which has similarities to Van der Pol oscillators. In this application, the idea is control the flow rate of the refrigerant fluid circulating in the microchannels and, consequently, the temperature at the walls of these channels, achieving better performance in heat transfer. Simulation results are presented to validate the applied control.

Keywords: Nonlinear System, Synchronization, Oscillators, Thermal Management.

1. INTRODUCTION

The Van der Pol oscillators is a nonlinear system that is often used to model phenomena in several fields of science, such as physics, biology (Kaplan et al. 2008), and engineering. Some applications include circadian rhythms, ECG signals (Kaplan et al. 2008), among others related to the electrical signals of the heart. One of the characteristics of this system is that it has a stable limit cycle. All trajectories in the vicinity of this limit cycle tend to move towards this oscillation.

In this article, the goal is stabilize a set of identical Van der Pol oscillators (connected in parallel) applying a single input control. To perform this objective, it is necessary to synchronize the oscillators in phase (meaning they are in the same initial conditions), as it is not possible to control identical systems in parallel with only a single input control applied to all of them.

One aplication for this control is the micro-thermal-fluid cooling system. With the advancement of technology, electronic applications increasingly require cooling systems with better performance and greater efficiency (Garimella et al. 2008). Thermal-fluid cooling systems with microchannels have emerged as a great alternative to control temperature in high-power electronic devices such as computer processors, GPUs, and other advanced electronics components. These systems consist of a series of microscopic channels filled with a heat transfer fluid, which is used to remove excess of heat generated by the electronic device.

These systems are highly energy efficient and are capable of removing large amounts of heat (Lee and Mudawar 2009), compared to more traditional cooling systems such as fan-cooled heat sinks.

Although highly efficient, they also exhibit some instabilities that are challenges for their performance, such as uneven flow distribution and pressure drops in the channels (Oevelen et al. 2017). This limits the amount of heat flow that can be dissipated, inducing a temperature increase at the heat source, decreasing the system's performance (Jin et al. 2022a).

Synchronizing the flows in the channels maximizes heat transfer efficiency by reducing the formation of dead zones, areas where the fluid is not flowing properly and therefore not transferring heat efficiently. The formation of dead zones or non-uniform fluid distribution can significantly reduce the cooling system's performance, leading to an increase in electronic device temperature and reducing its lifespan.

To achieve this synchronization, it is considered that the oscillations presented by this system can be modeled as Van der Pol oscillations (Lizarralde et al. 2017, Jin et al. 2022a and Zhang et al. 2010). It is applied an input signal (equal in the channels) that oscillates according to the phase behavior of the systems, as seen in (Bai and Wen 2019).

The control law applied in (Bai and Wen 2019) introduces the concept of the phase response curve (PRC), which is a characteristic curve of oscillators. It is a response function to small perturbations applied to the oscillatory system, resulting in a variation of the phase (Cestnik and Rosenblum 2018).

In comparison to the control proposed in this work, the one applied in (Jin et al. 2022b) is considered. In the experiment, the author investigates the effect of oscillating amplitude and frequency of the input valve in the system.

⋆ This study was financed in part by Coordenação de Aperfeiçamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001 e 88887.136349/2017-00, FAPERJ and CNPq.

At a certain amplitude and frequency, the synchronization of flow rates and temperatures in the channels is verified, mitigating the problems of flow maldistribution in the channels, temperature variations, and improving the system's performance.

Finally, with the synchronized channels, a feedback control is designed to stabilize the system and ensure that the temperature in the channels remains below a predetermined limit.

2. VAN DER POL MODEL

The second-order nonlinear differential equation that describes the Van der Pol system is given by

$$\ddot{x} - \mu\ (1 - x^2)\ \dot{x} + x = 0, \mu > 0 \tag{1}$$

where μ is the damping parameter.

This system has an unstable equilibrium point at $x = 0$ and a stable limit cycle. All trajectories ($t \to \infty$) converge to this cycle, given an initial condition different from the origin.

Let us consider that we have N parallel oscillators with the characteristics described above, with identical parameters. The goal is to control these N oscillatory systems, applying only a single input.

The systems can be characterized as follows, in state-space equations,

$$\begin{aligned} \dot{x}_{1i} &= x_{2i} \\ \dot{x}_{2i} &= \mu\ (1 - x_{1i}^2)\ x_{2i} - x_{1i} + u \end{aligned} \tag{2}$$

where x_{1i} and x_{2i} are the states of the $i - th$ system and $i = 1...N$, $\forall N > 0$.

3. CONTROL DESIGN

To achieve the control of those oscillatory systems, the first step is the synchronization. This is a crucial step to achieve the stabilization, since identical systems are non-controllable, considering parallel interconnection with single input control. When identical systems are synchronized, it represents they are in the same conditions, making it possible to stabilize all of them.

3.1 Synchronization Control

A single input control is applied to the Van der Pol oscillators, aiming to synchronize them. For this, it is used the synchronization control presented in (Bai and Wen 2019).

In (Bai and Wen 2019) is used an oscillator as a reference for the systems to be synchronized. This reference oscillator has a natural frequency ω and its phase dynamic depends on the phase of the oscillators we want to synchronize. Its dynamic is given by:

$$\dot{\theta}_a = \omega + \gamma \sum_{i=1}^{N} \sin(\theta_i - \theta_a), \gamma > 0 \tag{3}$$

where θ_i is the phase of the i-th oscillator and θ_a is the phase of the reference oscillator. The parameter γ is a gain coefficient for adjustment.

The control law applies the concept of phase response curve (PRC), which is a characteristic of each oscillatory system. This curve describes how much the system's phase varies given a perturbation applied to it. To perform the synchronization control, it is used the PRC function of a Van der Pol oscillator, as depicted in Figure 1 (based on Cestnik and Rosenblum 2018).

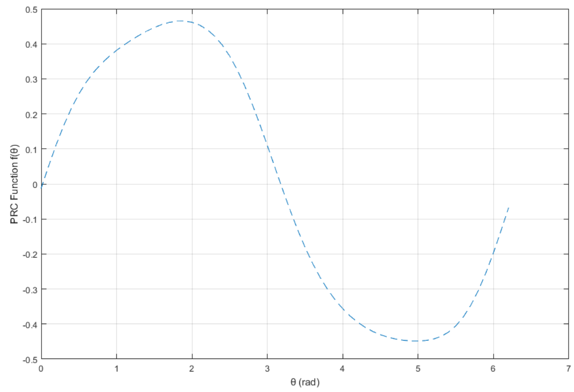

Fig. 1. Van der Pol PRC Function.

As described in (Bai and Wen 2019), the control law is given by

$$u = -k\ \sin(\delta^T)\ f(\theta) \tag{4}$$

where u is a scalar, meaning that a single control input will be applied to the systems, δ is the phase difference (phase error) between the oscillators and the reference oscillator, and $f(\theta)$ represents the PRC function of the oscillators.

$$\delta = (\theta_1, ..., \theta_N)^T - \theta_a 1_N = (\delta_1, ..., \delta_N)^T \tag{5}$$

To determine the phase of a Van der Pol oscillator, it is considered that, with the limit cycle around the origin, the approximation $\arctan(\dot{x}/x)$ is a valid proposition.

The control (4) ensures the synchronization of the oscillators for almost all initial conditions, and it is proved (Bai and Wen 2019) that the equilibrium point $\delta_i = 0$ is almost globally asymptotically stable, since the undesirable equilibrium points in the closed-loop system are unstable.

For a bounded control, control law (4) is modified to

$$u = -u_b\ \rho_u\ (sin(\delta^T) f(\theta)) \tag{6}$$

where $u \in [0, u_b]$, $\forall u_b > 0$. And ρ_u is a continuous function that ensures the boundedness of u (Bai and Wen 2019).

In figure 3, there is the synchronization control model diagram proposed in this paper.

In (Jin et al. 2022b) uses an open-loop control to achieve the synchronization of two oscillatory systems. It is a cooling system with microchannels, where a certain frequency and amplitude of opening and closing the valve, which controls the input flow rate of a refrigerant fluid, leads to synchronization of the fluid flows and temperatures on the walls of the channels. By varying this valve (represented by a square wave), the systems synchronize in amplitude and phase.

Here, to achieve the same synchronization as described above, it is used a closed-loop control. In the simulation section, it is demonstrated the open-loop control described above in comparison with the closed-loop control.

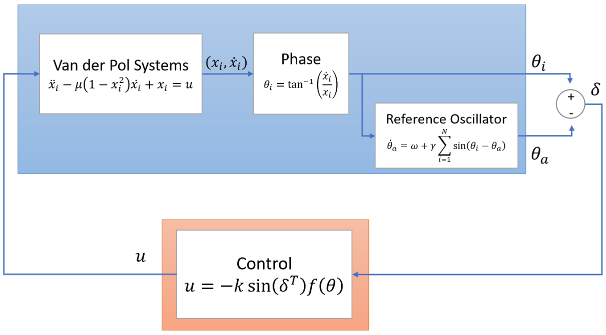

Fig. 2. Synchronization Control Model Diagram.

3.2 Controller Design

Stabilizing the systems is the next step after achieving synchronization. As the oscillators are in phase, we can apply a single control input to bring the systems to a specific point.

Since all subsystems are in phase, we can apply a single input to all oscillators, given by

$$u = -k\ \dot{x}_{11} = -k\ x_{21} \tag{7}$$

where x_{21} is a state variable of the first oscillator.

The objective is the stabilization of the system applying control law in (7), making the origin asymptotically stable.

Theorem 1. Consider the dynamics of the oscillators in (2) with the control law in (7) and a equilibrium point at the origin. For gain $k > \mu$, for all $\mu > 0$, the equilibrium point becomes globally asymptotically stable.

Proof. As the oscillators are synchronized, it is sufficient to prove that control (7) works for one oscillator. To prove stability, we can adopt as a Lyapunov function

$$2\ V(x_{11}, x_{21}) = x_{11}^2 + x_{21}^2 \tag{8}$$

where x_{11} and x_{21} are the state variables of the Van der Pol system. The function $V(x_{11}, x_{21})$ is positive definite, since $V(0,0) = 0$ and $V > 0$, $\forall x_{11}, x_{21} \neq 0$. It is also a radially unbounded function, thus complying with the Direct Method of Lyapunov.

Following direct method, we will derive the Lyapunov function based on equations (2) and (7).

$$\begin{aligned}\dot{V}(x_{11}, x_{21}) &= x_{11}\dot{x}_{11} + x_{21}\dot{x}_{21} \\ &= x_{11}x_{21} + \mu(1 - x_{11}^2)x_{21}^2 - x_{11}x_{21} + ux_{21} \\ &= x_{21}^2[(\mu - k) - \mu x_{11}^2]\end{aligned} \tag{9}$$

The derivative of the candidate Lyapunov function is semi-definite negative, for $k > \mu$, for all $\mu > 0$. However, applying LaSalle's invariance principle, for $x_{21} = 0$, we have $\dot{x}_{21} = 0$ and, therefore, $x_{11} = 0$. We can conclude that the applied control (7) makes the origin a globally asymptotically stable equilibrium point.

It is possible to apply a change of variables to steady-state error $e = x - x_d$, to make the system reach a desired equilibrium point x_d. ■

3.3 Observer Design

The state x_2 is not easily measured, because the real system is contaminated by noise. For this reason, we need to design a state observer to estimate the states of (2). The state-space representation of the system is given by:

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -1 & f(x_1)\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}u \tag{10}$$

where $f(x_1)$ is the nonlinear part $\mu\ (1 - x_1^2)$.

Theorem 2. For the observer, we have

$$\begin{bmatrix}\dot{\hat{x}}_1 \\ \dot{\hat{x}}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -1 & f(x_1)\end{bmatrix}\begin{bmatrix}\hat{x}_1 \\ \hat{x}_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}u + \begin{bmatrix}L_1 \\ L_2\end{bmatrix}(x_1 - \hat{x}_1) \tag{11}$$

considering that x_1 is the system's output and $[L_1 L_2]^T$ are the observer gains. The states are asymptotically estimated by the observer if the gains satisfy the conditions in (12).

$$\begin{aligned}L_1 - f(x_1) > 0 \\ L_2 + 1 - L_1 f(x_1) > 0\end{aligned} \tag{12}$$

Proof. The observer system can be rewritten in the states equation below.

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + L_1(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= \mu(1 - x_1^2)\hat{x}_2 - \hat{x}_1 + L_2(x_1 - \hat{x}_1) + u\end{aligned} \tag{13}$$

Let $\tilde{x}_1 = \hat{x}_1 - x_1$ and $\tilde{x}_2 = \hat{x}_2 - x_2$ be the observer errors, subtracting (13) from (10), gives the estimator error.

$$\begin{aligned}\dot{\tilde{x}}_1 &= \tilde{x}_2 - L_1\tilde{x}_1 \\ \dot{\tilde{x}}_2 &= \mu(1 - x_1^2)\tilde{x}_2 - \tilde{x}_1 - L_2\tilde{x}_1\end{aligned} \tag{14}$$

We have the same representation above in matrix form:

$$\begin{bmatrix}\dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2\end{bmatrix} = \begin{bmatrix}-L_1 & 1 \\ -L_2 - 1 & f(x_1)\end{bmatrix}\begin{bmatrix}\tilde{x}_1 \\ \tilde{x}_2\end{bmatrix} \tag{15}$$

The characteristic equation of the observer error system is:

$$\lambda^2 + (L_1 - f(x_1))\lambda + (L_2 + 1 - L_1 f(x_1)) = 0 \tag{16}$$

The estimator error is stable if and only if the conditions (12) are satisfied.

We can conclude that the states of the system can be asymptotically estimated by the observer (11) if the gains L_1 and L_2 are chosen according to (12). ■

4. FLOW MICRO-THERMAL-FLUID COOLING SYSTEM

In Figure 3, a cooling system from (Zhang et al. 2010) is illustrated, where an artificial surge tank is located upstream the flow boiling channels to characterize the lumped of the compressible volumes. Additionally, two virtual restriction elements are placed to represent the flow resistance from the pump to the main boiling branch and the surge tank. In the schematic, the boiling channel represents all the microchannels system, since pressure fluctuations are measured between the inlet and outlet of the microchannels. Notice that the flow meter model was placed before the boiling channel because the pressure drop across microchannels is significant and cannot be disregarded.

In (Lizarralde et al. 2017), the mathematical model of flow rate in a micro-thermal-fluid cooling system shows similarities with the Van der Pol oscillator. The same similarities are pointed out in Jin et al. 2019, where the author compares the characteristics of pressure drop oscillation with Van der Pol equation. The nonlinearity of

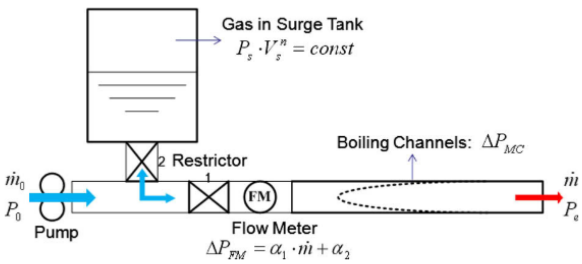

Fig. 3. Schematic of boiling channel with upstream surge tank

the model represents the variation in pressure drop at the outlet and inlet of the microchannels. Considering these channels are approximately identical, it is not possible to stabilize the flow with a single input control. This problem can be overcome if the systems are in the same conditions, i.e., synchronized in phase and amplitude. This means that a control law can be applied to stabilize the systems and reach a specific operation point.

As described in (Lizarralde et al. 2017), the mathematical model of the flow is given by:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -f(z_1, q)z_2 - dz_1 + du \end{aligned} \tag{17}$$

where z_1 is the flow rate and the parameter d is a positive constant. Considering $f(z_1, q)$ as the nonlinearity of the system, we can assume that it is in the form $\mu(1 - z_1^2)$. Therefore, the system can be modeled as a Van der Pol oscillator (Lizarralde et al. 2017) and we can apply the synchronization-based control analyzed in the previous section.

The dynamics for the temperature in the walls of the microchannels is characterized in (Lizarralde et al. 2017) by:

$$\dot{z}_3 = -k_1 S_w g(z_1, q, u) z_3 + k_1 q \tag{18}$$

where z_3 is the temperature at the channel wall minus the fluid temperature, and $g(z_1, q, u)$ is the heat transfer coefficient (HTC), which strongly depends on the fluid flow rate and has the characteristic shown in Figure 4.

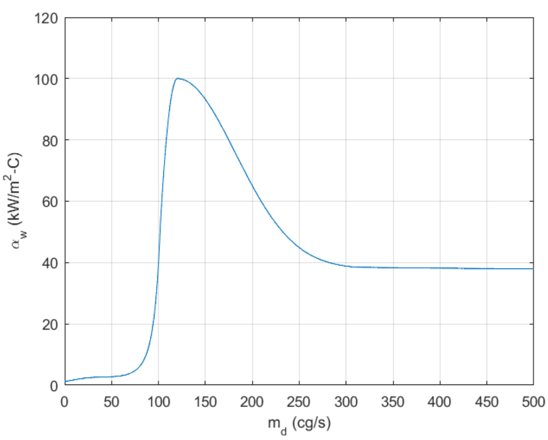

Fig. 4. Heat transfer characteristics versus mass flow rate $(G = \dot{m}/A)$

Instabilities in the flow through the channels, such as pressure drop oscillation and uneven flow distribution, can cause an uneven temperature variation and affect the system's performance (Jin et al. 2022a). The objective of flow synchronization is mitigate these instabilities and make the temperature in the channels vary equally. With this, we can stabilize the system and bring the temperature in the channel walls to a desired value (below a specific level $z_3 < z_{3max}$).

In (Jin et al. 2022b), the author analyzes the effect of opening and closing the control valve of the input flow on the fluid flow and wall temperatures. It is experimentally verified that a certain amplitude and frequency of oscillation of the input valve results in synchronization of flow rate and temperature in the channels. It is an open-loop system, where the input is manually adjusted until an output result is achieved.

In this paper, the system operates in a similar way, but with a feedback control, since there is a control law that determines the input signal. This variation results in synchronized flow and temperature signals.

5. SIMULATIONS

Initially, we apply the synchronization control (6), based on (Bai and Wen 2019), to five Van der Pol oscillators ($N = 5$), to check results and performance. We proceed with simulations for the cooling system application, with two microchannels: open-loop control, following the approach described in (Jin et al. 2022b), and then closed-loop control with the PRC. MATLAB and Simulink were used to simulations.

5.1 Synchronization Control - Van der Pol

First, we apply the control law (6) to five Van der Pol oscillators ($N = 5$), and then check the results in figures 5, 6 and 7.

The parameters for these simulations were: $\mu = 2$, $\gamma = 22$ and $\omega = 2$. Initial conditions were randomly generated.

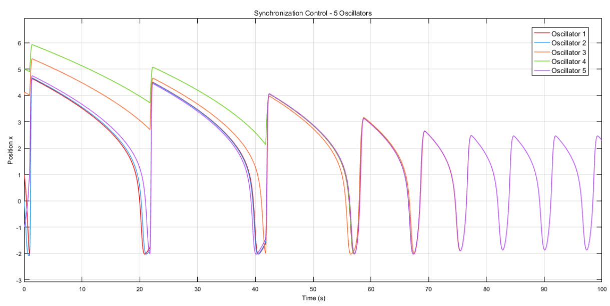

Fig. 5. Output signals (Synchronization Control - Van der Pol).

The Root Mean Square Difference (RMSD) is used as a quantitative measure of synchronization, i.e. a measure of the average difference between oscillator's phases. In this simulation, as illustrated in figure 7, the RMSD converges to zero, validating the efficiency of synchronization.

5.2 Open-loop Synchronization

Applying a square wave with mean value 0.6, amplitude of 0.3, and frequency 0.04 Hz to the input signal (system's

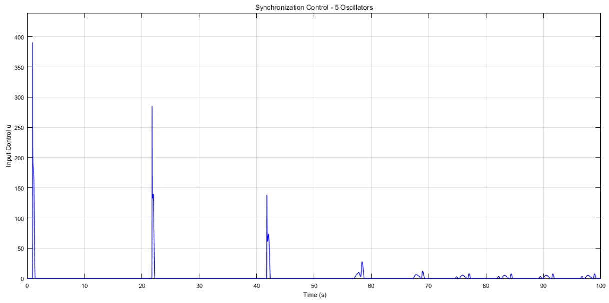

Fig. 6. Control input (Synchronization Control - Van der Pol).

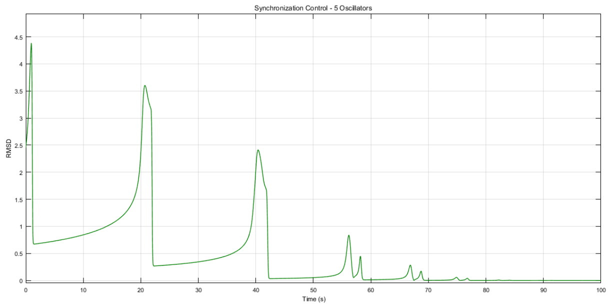

Fig. 7. RMSD (Synchronization Control - Van der Pol).

inlet flow rate), the output and the RMSD between the signals were verified.

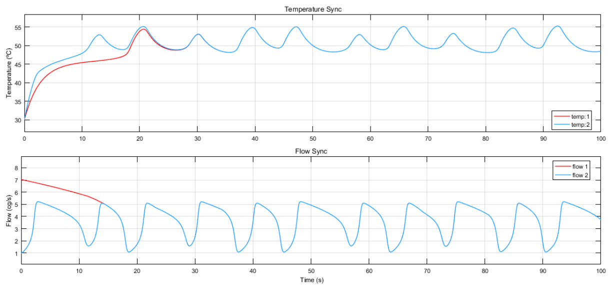

Fig. 8. Output signals (temperature and flow rate in two microchannels) - open-loop control.

The output signals (figure 8) appear to be synchronized. The mean value of RMSD for last 20 seconds of simulation, for the open-loop control, is 1.3e−3.

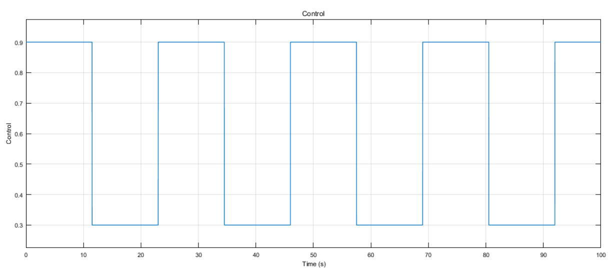

Fig. 9. Control input for synchronization (open-loop control).

5.3 Closed-loop Synchronization

We consider a system with two microchannels. The PRC function used in the control is shown in Figure 1 (based on

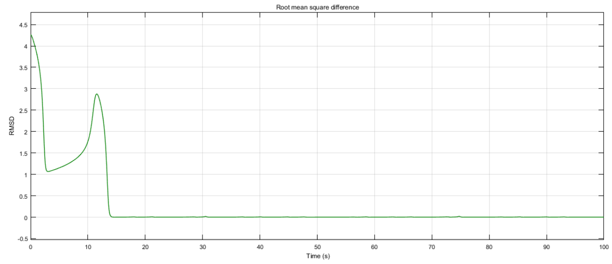

Fig. 10. RMSD for synchronization control (open-loop control).

Cestnik and Rosenblum 2018). For the reference oscillator, the natural frequency is $\omega = 2rad/s$, and the damping coefficient of the Van der Pol oscillators is $\mu = 2$. The initial conditions were randomly generated.

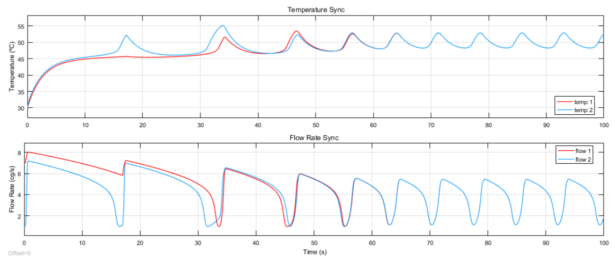

Fig. 11. Output signals (temperature and flow rate in two microchannels) - closed-loop control.

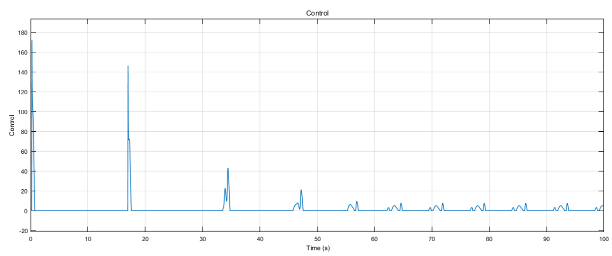

Fig. 12. Control input for synchronization (closed-loop control)

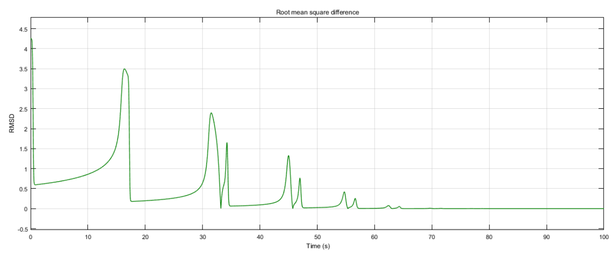

Fig. 13. RMSD for synchronization control (closed-loop control)

The mean value of RMSD for last 20 seconds of simulation is 1.7e−5, for closed-loop control. Compared to the open-loop system, the control (6) presents a significant improvement in performance for signal synchronization. Note that the principle is quite similar, as an oscillating signal is applied to the input of the system, but in this case it is used a characteristic curve (PRC) of the flow oscillation as the control signal of the valve.

5.4 State Estimations

For the observer system, gains L_1 and L_2 were considered equal to 10 and 100, respectively. The initial conditions were randomly chosen.

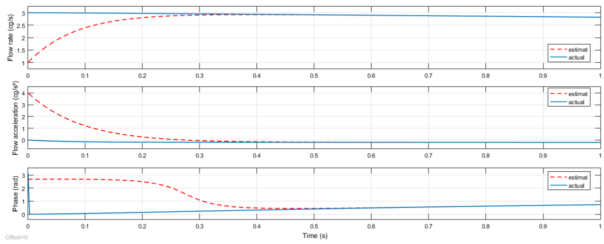

Fig. 14. State Estimations (top: flow rate; middle: flow acceleration; bottom: phase)

Satisfactory state estimation transients have been obtained within 0.5s, as depicted in figure 14.

5.5 System Stabilization

After closed-loop synchronization, the next step is to stabilize the flow rates and, consequently, the temperatures on the channel walls. To do so, let us use the control described in equation 7. As the signals are in phase, we can apply a single input to both systems.

As depicted in figure 15, we can verify the signals are synchronized and, at a certain moment, the control to stabilize the system is applied, guiding the outputs to a setpoint.

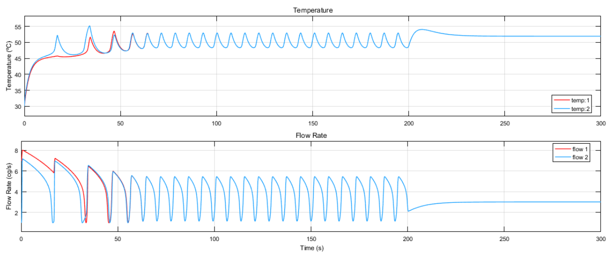

Fig. 15. Temperature and flow rate stabilization in two microchannels (system stabilization).

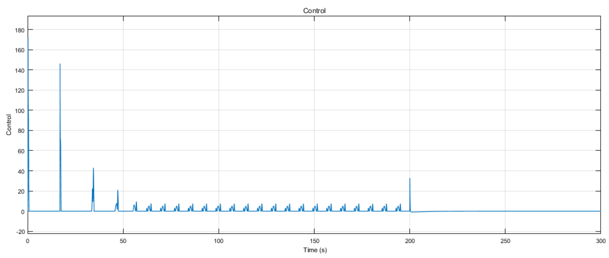

Fig. 16. Stabilization input control (system stabilization).

6. CONCLUSIONS

In this paper, a closed-loop control is proposed to stabilize a set of Van der Pol oscillators in parallel, applying only a single input to all of them. To make it possible, a synchronization process is previously applied. Furthermore, a nonlinear observer design is necessary as the state is not easily measured and the real system is contaminated by noise.

This proposed synchronization control is applied to micro-thermal-fluid cooling system and compared to the open-loop control proposed in Jin et al. 2022b. By using the root mean square deviation (RMSD) as a comparison parameter, it is concluded that closed-loop control law yields good results. With that, it is possible to mitigate fluid flow fluctuations in the channels and improve heat transfer efficiency in the system.

However, implementing synchronization control techniques requires a deep understanding of the system dynamics and a careful selection of control parameters, such as synchronization frequency and control signal amplitude. The simulations for the proposed control demonstrated its effectiveness and validated its efficiency in a current thermal management application.

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