Control of a thickening process based on a data-driven predictive controller ⋆

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Abstract: The mineral industry is comprised of several large-scale, complex processes that require tight control in order to operate appropriately. Among them, the thickening process is a solid-liquid separation unit operation whose highly nonlinear and slow dynamics pose challenges in obtaining an accurate process model. Consequently, model-based controllers, such as the model predictive control (MPC), despite all its advantages, do not achieve their best performance in such an industrial environment. In this work, we investigate using a data-driven predictive control (DDPC) approach to control the thickening process, in which we integrate a predictive control formulation and a prediction technique called Lazily Adaptive Constant Kinky Inference (LACKI). The proposed method makes use of process data and a machine learning technique to supply the lack of an accurate model. Simulated results show that this approach performs satisfactorily in controlling the thickening process.

Keywords: Data-driven control; model predictive control; machine learning; thickening process; mineral industry.

1. INTRODUCTION

A mineral plant is a facility that uses several interconnected, large-scale processes to treat the ore to make it suitable for the market. Among these processes, thickening is a solid-liquid separation process intended to create a high-content solid slurry by removing water added in previous stages. More than production purposes, thickening has an essential role in environmental issues. By recovering water, this is reused in other mineral processes, resulting in less consumption of fresh water, in addition to the fact that less water is sent to tailing dams, which may pose environmental and social risks (Jiao et al., 2021).

In order to better operate and seek improvements for the thickening process, a model of it would be extremely useful. However, as with any mineral process, the thickening process's inherent complexity combined with the randomly varying characteristics of the ore make it difficult to define a common ground model (King, 2001). As a result, proposed models often rely on simplifications, such as on equipment configuration (Betancourt et al., 2014; Tan et al., 2015), critical variables use (Usher and Scales, 2005), and dynamic material property changes (Nasser and James, 2007). Consequently, these models are prone to poorer correlations between real dynamics if applied to industrial equipment.

Another drawback of the lack of an accurate model of the thickening process is that the effectiveness of controlling it with model-based control techniques is jeopardized. Model predictive control (MPC) is a class of algorithms that, besides being able to act based on the predicted behavior of the system, stands out for other characteristics, such as robustness to multivariable systems, ability to handle process constraints, and capacity to keep the process close to operating limits (Camacho and Alba, 2013). Nonetheless, it relies heavily on an accurate model of the process it is controlling.

Tan et al. (2015) have evaluated by simulation the application of a linear MPC in a first principle-based model of a paste thickening process. The controller acts over the underflow rate to control the underflow solids concentration. The thickener feeding flow rate and solids concentration have been considered as disturbances to the process. The authors have conducted setpoint tracking tests from which they have concluded that the MPC approach has successfully controlled the process variables while maintaining the manipulated and secondary variables within predefined constraints. Later, the same authors have proposed using the same strategy to control a model of the thickening process by considering the effects of the rake torque on the process, also being used as a constraint to the controller

⋆ This work was carried out with the support of the the Brazilian agencies Coordination for the Improvement of Higher Education Personnel - Brazil (CAPES) through the Academic Excellence Program (PROEX), and National Council for Scientific and Technological Development (CNPq) under the grant 315695/2020-0.

(Tan et al., 2017). Few works are found in the literature regarding MPC applied to a modeled thickening process; works reporting industrial application, on the other hand, are rare.

Given the disadvantage of using conventional MPC structures for controlling a thickening process, one viable alternative would be using expert system strategies, such as Fuzzy logic, that do not rely on a model of the process to define its control action. Conversely, these define a set of rules built based on the understanding of the process dynamics. Works like Segovia et al. (2011); Bergh et al. (2015); Magalhães and Euzébio (2018) have proposed the use of expert systems to control either simulated or industrial thickening processes. However, despite the capacity to deal with processes whose model is not available, the effectiveness of using an expert system approach for a thickening process might not meet the potential of using MPC, as the former does not have the features that make the latter the most widely used form of advanced control applied in the process industry (Olivier and Craig, 2017; Rogers et al., 2019).

Thus, the alternative for applying MPC to the thickening process lies in the data-driven predictive control (DDPC). This controller belongs to the class of data-driven control - in literature, also referred to as data-based control, model-free control, or learning-based control - which is a class of control techniques that uses process data as inputs to contribute to the controller tuning procedure, decision-making, performance evaluation, model identification, and fault diagnosis (Hou and Wang, 2013).

Data-driven predictive control, then, integrates conventional MPC formulation with the learning capabilities of a machine learning (ML) technique. Currently, the main focus is on using ML to learn the dynamic behavior of a process based on historical data. That way, instead of using a process model, the ML technique predicts the future output behavior during the optimization of control action by the predictive control formualtion.

To what concerns DDPC applied to the thickening process, few works have been found in the literature. Núñez et al. (2019) have developed a neural network-based model predictive control (NNMPC) strategy to control a paste thickening process. A recurrent neural network (RNN), based on an encoder-decoder architecture, have been built to characterize the process and predict its behavior. Instead of building a single MIMO (multi-input multi-output) model, the authors have developed three independent MISO (multi-input single-output) models. Each of them has been trained firstly with historical data of the process and, later, during the online operation. Another stage of training has been conducted to take into account the time-varying aspects of the process. Two tests in an industrial facility have validated the NNMPC capacity to perform setpoint tracking despite strong disturbances. The authors have claimed that although the proposed method has yielded good results, the complexity of the neural network has imposed limitations on the controller updating rate. Thus, the same authors have proposed another data-driven controller for the same paste thickening process (Diaz et al., 2021). This time, they have used a Random Forest technique to model the process and make predictions upon the controlled variables. The RF-MPC proposed by the authors have been compared to a conventional MPC based on a benchmark ARIMAX model in a simulated environment. The former has showed better performance, both qualitatively and quantitatively.

In this work, we propose the use of MPC with a machine learning technique called Lazily Adaptive Constant Kinky Inference (LACKI) for controlling a thickening process. Unlike other ML techniques that have been already integrated with MPC for thickening control, the LACKI technique stands out for quantifying the uncertainty associated with each prediction; thus, allowing for improved robustness of the process control. By acting robustly, the controller can ensure that the process variables are maintained within limits despite uncertainty on predictions, avoiding production and economic losses.

The remainder of this work is organized as follows. In Section 2, we present the thickening process mathematical model. In Section 3, the DDPC formulation is thoroughly presented. Results of the control application are shown in Section 4. Finally, Section 5 concludes the work.

2. THE THICKENING PROCESS

Utilizing an enormous quantity of water is necessary for most mineral processes to properly beneficiate the ore. Consequently, at the end of the beneficiation chain, the water must be separated from the pulp - i.e., slurry composed of solid particles and water. This separation is carried out by a dewatering system whose main goals are: *i*) dry the ore particles to a point in which they are easy and cost-effective to transport; and *ii*) reduce fresh water consumption by recycling the recovered water (Chaedir et al., 2021).

The first unit operation of a dewatering system is the thickening process. It occurs within a thickener, such as the one depicted in Figure 2, that produces two output streams: a clarified overflow (recovered water) and a thickened underflow at a required concentration (Junior et al., 2022).

The solid-liquid separation is due to gravitational force triggered by the difference in the solid and liquid particle densities (Chaedir et al., 2021). The settling of solid particles produces regions whose solid concentrations are different. The top region is a clarified supernatant; an intermediate region, the settling region, in which the concentration is lower than a critical concentration; and the compaction region, where the concentration is larger than the critical concentration. Furthermore, flocculants are commonly added to the pulp to accelerate the particles' sedimentation rate (Wills and Finch, 2015).

2.1 Mathematical modelling

This work makes use of the model proposed in Pereira et al. (2020) as a computational environment for testing and validating the proposed controller. It combines features of two models previously developed in Betancourt et al. (2014) and Bürger et al. (2017). Whereas in Betancourt et al. (2014), the model incorporates the effects of reagent addition, the model proposed by Bürger et al. (2017)

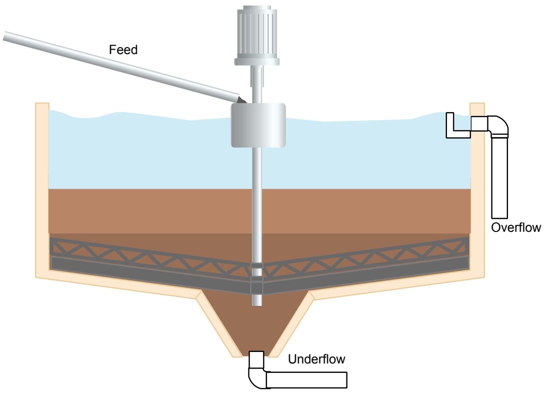


Figure 1. Schematic of the thickener.

brings a more faithful representation of the thickener geometry, depicted in Figure 2.

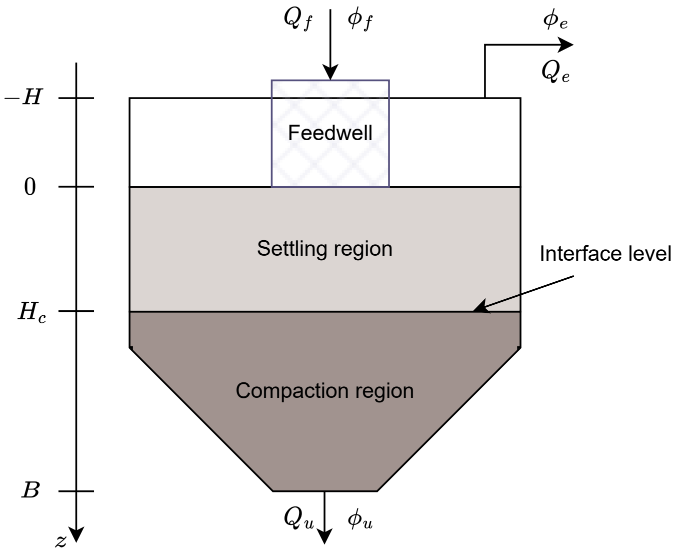


Figure 2. Schematic of the one-dimensional model of a thickener.

From Figure 2, the common operation of the thickener is described. A feeding flowrate, Q_f, enters the equipment by a feedwell with a concentration of ϕ_f. At the same time, the flocculant is added to the feeding flowrate. As the solid particles settle at the bottom, water is recovered as effluent at a flowrate, Q_e, with a solid concentration of ϕ_e, ideally low; and the thickened material is discharged as underflow at a flowrate of Q_u, with a concentration of ϕ_u. Moreover, the interface level, H_c, - i.e., the height that indicates the transition between the settling and compaction regions - is used as a metric of the quality of the thickener operation.

The model is comprised of non-linear partial differential equations (PDE) that describe the behavior of the process based on the depth, z, and time, t. Based on Betancourt et al. (2014), the model takes into account the addition of flocculants to the process, meaning that the sedimentation velocity of particles is affected by the flocculant dosage, $k(z,t)$. Then, the sedimentation velocity is given by

$$\overline{v_{hs}}(\phi, k) = k(z,t)v_{hs}(\phi), \tag{1}$$

in which $v_{hs}(\phi)$ is the particles sedimentation velocity with no flocculant addition. The compressibility of particles is, then, given by

$$\overline{d}(\phi, k) = \frac{k(z,t)v_{hs}(\phi)\sigma_e'(\phi)}{(\rho_s - \rho_f)g} = kd(\phi), \tag{2}$$

in which ρ_s and ρ_f are the solid and fluid densities, respectively; g is the gravity acceleration; and σ_e' is the solids' effective stress function derivative. Accordingly, the solid concentration, ϕ, and the feed flux of particles carrying property $k(z,t)$, w, are given by the following set of PDE:

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}\overline{F}(\phi, w/\phi, z, t) = \frac{\partial}{\partial z}\left(\gamma(z)\frac{w}{\phi}\frac{\partial D(\phi)}{\partial z}\right) + \frac{Q_f(t)\phi_f(t)}{A}\delta(z), \tag{3}$$

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial z}\left(\frac{w}{\phi}\overline{F}(\phi, k, z, t)\right) = \frac{\partial}{\partial z}\left(\gamma(z)\frac{w^2}{\phi^2}\frac{\partial D(\phi)}{\partial z}\right) + \frac{Q_f(t)w_f(t)}{A}\delta(z), \tag{4}$$

in which A is the thickener cross-sectional area; δ is the Dirac delta function that models the feed inlet at $z = 0$; γ is a parameter that indicates if z is a depth inside ($z = 1$) or outside ($z = 0$) the equipment; and $\overline{F}$ incorporates a function of the in and out flowrates. The value of $\overline{F}$, given the different regions of operation, is defined by

$$\overline{F}(\phi, z, t) = \begin{cases} -Q_e(t)\phi/A, & z < -H, \\ -Q_e(t)\phi/A + kv_{hs}(\phi)\phi, & -H < z < 0, \\ Q_u(t)\phi/A + kv_{hs}(\phi)\phi, & 0 < z < B, \\ Q_u(t)\phi/A, & z > B. \end{cases} \tag{5}$$

To enhance the faithfulness of the model to real equipment, Pereira et al. (2020) have incorporated the variation of the cross-sectional area presented in Bürger et al. (2017). Therefore, the model used in this work considers a cross-sectional area vector, $A(z)$, defined for each of the three cross-sectional distinct regions as follows

$$A(z) = \begin{cases} \frac{\pi}{4}(D_{max}^2 - D_{fw}^2), & -H < z \le 0, \\ \frac{\pi}{4}D_{max}^2, & 0 < z \le H_c, \\ \frac{\pi}{4}D(z)^2, & H_c < z \le B, \end{cases} \tag{6}$$

in which $D(z)$ is the diameter of the thickener, with linear variation between $H_c < z < B$. It is considered $\{D(z) = D_{max} | z \ge H_c\}$, and $\{D(z) = D_{min} | z = B\}$; D_{fw} is the diameter of the thickener feed pit.

Now, given that the cross-section area is variable and dependent on z, Equations (3) and (4) are rewritten as

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}\overline{F}(\phi, w/\phi, z, t) = \frac{\partial}{\partial z}\left(\gamma(z)\frac{w}{\phi}\frac{\partial D(\phi)}{\partial z}\right) + \frac{Q_f(t)\phi_f(t)}{A(z)}\delta(z), \tag{7}$$

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial z}\left(\frac{w}{\phi}\overline{F}(\phi, k, z, t)\right) = \frac{\partial}{\partial z}\left(\gamma(z)\frac{w^2}{\phi^2}\frac{\partial D(\phi)}{\partial z}\right) + \frac{Q_f(t)w_f(t)}{A(z)}\delta(z). \tag{8}$$

Defining $Q(t)$ as

$$Q(t) = \begin{cases} -Q_e(t), & z < 0, \\ Q_u(t), & z > 0, \end{cases} \tag{9}$$

function $\overline{F}(\phi, k, z, t)$ is redefined to

$$\overline{F}(\phi, k, z, t) = \begin{cases} Q(t)\phi/A(z), & z < -H, \\ Q(t)\phi/A(z) + kv_{hs}(\phi)\phi, & -H < z < B, \\ Q(t)\phi/A(z), & z > B. \end{cases} \tag{10}$$

Finally, multiplying (7) and (8) by $A(z)$ yields the following set of PDEs that represent the thickener:

$$\frac{\partial(A(z)\phi)}{\partial t} + \frac{\partial}{\partial z}\left(A(z)\overline{F}(\phi, w/\phi, z, t)\right) = \frac{\partial}{\partial z}\left(A(z)\gamma(z)\frac{w}{\phi}\frac{\partial D(\phi)}{\partial z}\right) + Q_f(t)\phi_f(t)\delta(z), \tag{11}$$

$$\frac{\partial(A(z)w)}{\partial t} + \frac{\partial}{\partial z}\left(\frac{w}{\phi}A(z)\overline{F}(\phi, k, z, t)\right) = \frac{\partial}{\partial z}\left(A(z)\gamma(z)\frac{w^2}{\phi^2}\frac{\partial D(\phi)}{\partial z}\right) + Q_f(t)w_f(t)\delta(z). \tag{12}$$

3. LACKI-BASED DATA-DRIVEN PREDICTED CONTROL

3.1 Lazily Adaptive Constant Kinky Inference

Lazily Adaptive Constant Kinky Inference (LACKI) is a non-parametric learning and inference method based on knowledge about boundedness, Hölder continuity, and error bounds on observations and inputs (Calliess, 2014). Also, LACKI is a technique that favors conservative decision-making as it never underestimates the true uncertainty associated with the predictions. Given this characteristic, its integration with MPC formulation has been discussed in Limon et al. (2017); Manzano et al. (2019, 2020, 2021).

Such as any supervised machine learning method, LACKI makes use of a dataset of a process to build a function that can generalize and predict the process output given unobserved inputs. As a single finite dataset can have infinite ways of being explained, the procedure of constructing the model depends on prior assumptions that define how the function generalizes over the dataset. For the LACKI method, the assumption is that the ground truth function - i.e., the function that dictates the true dynamics of the process - is assumed to be contained in a prior hypothesis space, K_{prior}, whose members are Hölder continuous functions, which may be contained within predefined bounds.

A Hölder continuous function is a generalization of a Lipschitz function. The former is defined by

$$\sigma_y(f(x), f(x')) \leq L(\sigma_x(x, x'))^p, \tag{13}$$

in which x and x' are two points contained in the input space $\mathcal{X}$; σ_y and σ_x are pseudo-metrics that indicate the distance between two elements; and p and L are constants, such that $\{p, L\} \geq 0$. This same definition is addressed to Lipschitz functions when $p = 1$, from which L is referred to as Lipschitz constant.

The procedure of learning a ground truth function $f : \mathcal{X} \to \mathcal{Y}$ - in which $\mathcal{X}$ and $\mathcal{Y}$ are the input and output spaces, respectively - implicates in forming a posterior belief such that,

$$f \in K_{post} \supseteq K_{prior} \cap K(D), \tag{14}$$

in which $K(D)$ is the set of all functions that could have generated the dataset D. From (14), one can notice that the conservatism of the method lies in the fact that no function that could have generated the dataset is ruled out during the learning procedure.

Based on this concept, the LACKI technique establishes tractable inference rules that never underestimate the true uncertainty associated with predictions. In other words, a predictor $\hat{f}_n : \mathcal{X} \to \mathcal{Y}$ and an uncertainty quantifier $\hat{\sigma}_n : \mathcal{X} \to \mathbb{R} \geq 0$ are defined to make inferences given any query input $x \in \mathcal{X}$. Thus, let f be the ground truth function and D a dataset such that $D := \{(s_i, \tilde{f}_i, \varepsilon(s_i)) | i = 1, \ldots, N\}$. Each sample of D is composed of an input value, s_i, an output value, $\tilde{f}_i$, and an interval-bounded observational error, $\varepsilon(s_i)$. And let $\underline{B}$ and $\overline{B}$ be, previously specified, lower and upper bounds, respectively. For a given query input x, the jth components, for $j = 1, \ldots, m$, of the inference rules are given by

$$\hat{f}_{n,j}(x) = w_u \min\{(\bar{B}_j(x), u_{n,j}(x; L(n))\} + w_l \max\{(\underline{B}_j(x), l_{n,j}(x; L(n))\}, \tag{15}$$

$$\hat{\sigma}_{n,j}(x) = w_u \min\{(\bar{B}_j(x), u_{n,j}(x; L(n))\} - w_l \max\{(\underline{B}_j(x), l_{n,j}(x; L(n))\}, \tag{16}$$

in which w_u and w_l are function weights such that $w_u(x), w_l(x) \in [0, 1]$ and $w_u(x) + w_l(x) = 1$. Functions u_n, l_n are, respectively, the ceiling and floor functions, whose jth components are given by

$$u_{n,j}(x; L(n)) = \min_{i=1,\ldots,N_n} \tilde{f}_{i,j} + L_j(n)\sigma_x^p(x, s_i) + e_j(x), \tag{17}$$

$$l_{n,j}(x; L(n)) = \max_{i=1,\ldots,N_n} \tilde{f}_{i,j} - L_j(n)\sigma_x^p(x, s_i) - e_j(x), \tag{18}$$

in which L and p are specified in advance; $\sigma_x(x, s_i)$ is a pseudo-metric; and e is an error value that describes the confidence in the data used to construct the dataset D. The set of Equations (15) and (16) is referred to *kinky inference rule*.

The Hölder constant L, in most cases, is not available a priori. To quantify it, one must be careful because a value of L chosen too conservatively high yields predictions that are too steep, hence jeopardizing the method's performance. On the other hand, a value of L chosen too small can make the method unable to infer the real shape of the ground truth function (Calliess, 2014). Therefore, Calliess (2014) has proposed an approach to determine an initial guess for the Hölder constant that might be updated in the face of new information - i.e., a new sample is added to the dataset. This approach has received the name *Lazily Adaptive Constant*. The initial estimate for L is given by

$$L(n) = \max_{i,j=1,\ldots,N_n, \sigma_x(s_i,s_j)>0} \frac{\sigma_y(\tilde{f}_i, \tilde{f}_j) - 2\bar{e}}{\sigma_x^p(s_i, s_j)}, \tag{19}$$

with $\bar{e}$ being the maximum error contained in the dataset. In case the dataset is updated by the addition of a new sample, the value of constant L can be uptaded by

$$L(n+1) = \max\left\{L(n), \max_{i=1,\ldots,N_n} \frac{\sigma_y(\tilde{f}_{N+1}, \tilde{f}_i) - 2\bar{e}}{\sigma_x^p(s_{N+1}, s_i)}\right\}. \tag{20}$$

3.2 k-Nearest Neighbors

As LACKI is a non-parametric technique, the function constructed to generalize over observed data does not have a prior fixed number of parameters; instead, this quantity depends on the information contained in the dataset. The advantage is that few function assumptions are made a priori, giving the technique more flexibility to learn richer classes of predictors. Its drawback is that, in order to make a prediction, it must evaluate the whole dataset, which might impose challenges for predictions as the dataset gets larger.

To ease the computational burden of prediction for large datasets, Calliess (2014) has proposed the use of the k-nearest neighbors (kNN) technique to create a subset of samples from which the prediction would be made. This subset is query-dependent, meaning that it is composed of the k nearest samples of the query. This approach is valid given that the samples with inputs closer to the query tend to influence the prediction value more than samples containing distant input values.

Accordingly, the kNN method is a non-parametric supervised machine learning algorithm applied for classification and regression activities (Taunk et al., 2019). Given an input query, its searching procedure consists of evaluating the distance between the query and each dataset sample, by which, at the end of the procedure, the k-nearest neighbors are selected. For continuous data, the Euclidean distance method is the most used to calculate the distance (or similarity) between the query and dataset samples (Taunk et al., 2019). Therefore, it is used throughout this work. Its formulation is given by

$$d(q,x) = \sqrt{\sum_{i=1}^{n}(q_i - x_i)^2}, \tag{21}$$

in which d is the Euclidean distance between a query q and a sample x, and n is the number of attributes of the data.

3.3 LACKI Predictive Controller

The essence of the predictive control formulation is to solve a finite horizon optimal control problem at each sampling time once the prediction horizon gets receded and the optimization problem initial conditions get updated (Camacho and Alba, 2013). Besides acting based on a system's predicted future behavior, predictive control stands out for other characteristics, such as robustness to multivariable systems, ability to handle process constraints, and keeping process close to operating limits (Camacho and Alba, 2013).

The integration of predictive control with the LACKI technique means that the predicted outputs during the control sequence optimization are given by the latter. Hence, a constrained optimization problem is posed in the following form:

$$\min_{\Delta u} \quad V(y,u) \tag{22}$$

$$\text{s.t.} \quad \hat{y}(k) = \hat{f}(y(k-1), u(k-1)), \tag{23}$$

$$y_{min} \leq y \leq y_{max}, \tag{24}$$

$$\Delta u_{min} \leq \Delta u \leq \Delta u_{max}, \tag{25}$$

in which Equation (23) gives the predicted output of the process computed through Equation (15); Equations (24) and (25) are constraints on the process output and on the rate of change of the controller's control action, respectively; and $V(y, \Delta u)$ is the cost functional, given by

$$V = \sum_{i=0}^{p} ||y_{sp}(k+i|k) - \hat{y}(k+i|k)||_Q^2 + \sum_{i=0}^{m-1} ||\Delta u(k+i|k)||_R^2, \tag{26}$$

4. TESTS AND RESULTS

To evaluate the proposed controller, it was put to control a thickener represented by the aforementioned mathematical model. The manipulated variable is the underflow rate, Q_u, and the controlled variable is the density of the underflow. The underflow density is obtained through the underflow concentration, ϕ_u, and was chosen because it is more often considered during the operation of an industrial thickener. Moreover, a standard linear MPC was implemented. The thickener linear model was identified at the operating point around $2.03 g/cm^3$ given the input value of $300 m^3/h$, resulting in

$$\frac{Y(s)}{U(s)} = \frac{-2.314}{16857s + 1}, \tag{27}$$

with the time constant being given in seconds.

4.1 Obtaining the dataset

The workspace is bounded by $Q_u^{min} = 260 m^3/h$ and $Q_u^{max} = 340 m^3/h$. In order to construct the dataset with samples that represent the behavior of the process along the range of operations given different conditions of controlled and manipulated variables, a set of experiments were formulated. Each experiment takes the process to a steady-state condition given a different value of Q_u, which varies between $Q_u = 265$ and $Q_u = 335$ in steps of $0.2 m^3/h$. For each steady-state condition, the manipulated variable changes from ± 5 in steps of $1 m^3/h$. Figure 3 illustrates one of these experiments.

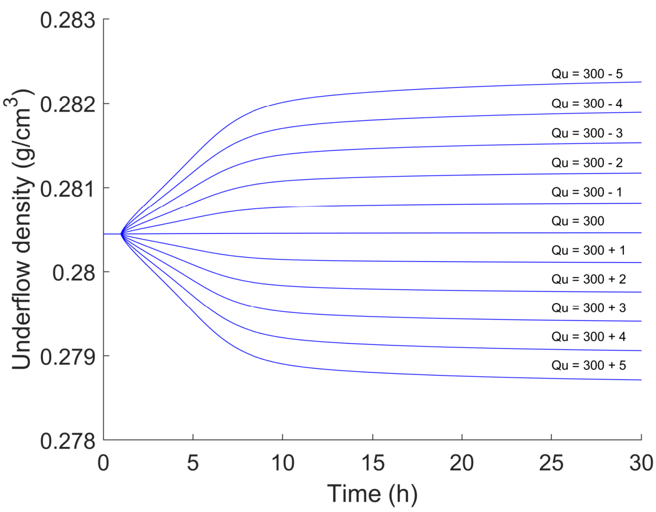


Figure 3. Experiment test responses used to build the dataset.

The sampling time used to collect the underflow concentration value was 2h. Hence, each sample of the dataset contains two inputs: the value of Q_u and the current underflow density; and one output: the underflow density two hours later than the instant in which the input values were recorded.

4.2 Control of the thickener

Both predictive controllers were validated in a setpoint tracking test. Whereas the standard linear MPC was tuned using weight matrices $Q = 8 \times 10^5$ and $R = 5 \times 10^5$, the DDPC controller used $Q = 5 \times 10^5$ and $R = 2 \times 10^8$. The parameters of the LACKI technique were set to $\overline{B} = Inf$, $\underline{B} = -Inf$, $p = 1$, $e = 0$, $w_u = w_l = 0.5$, $L = 1$. The number of samples selected for each prediction are $k = 50$. Furthermore, for both controllers, the prediction horizon, N_p, and control horizon, Nc, are equal to 5, and the the sampling period is equal to 2h. The optimization problem is solved in Matlab using the optimization function *fmincon*.

Figure 4 presents the behavior of the process for a setpoint tracking test. Both controllers managed to reach the setpoint. However, in some cases, the DDPC could not return the controlled variable to the setpoint after an overshoot before the following setpoint change. This might be due to the sampling time used for constructing the dataset that might be too long. Regardless, the largest error during the DDPC operation was about 0.5% of the setpoint variation.

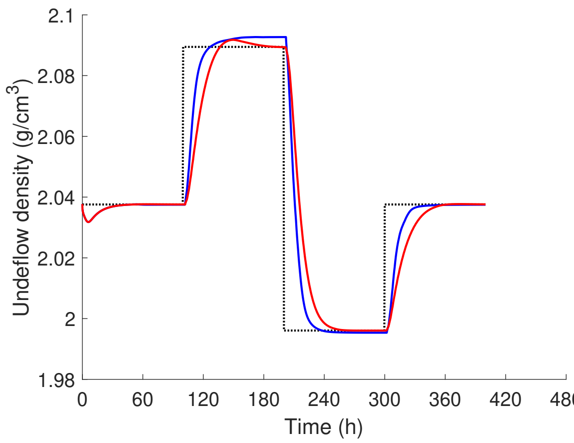
We compared the performance of both controllers using the integral squared error (ISE) metric, given by

$$ISE = \int_0^\infty e^2 dt. \tag{28}$$

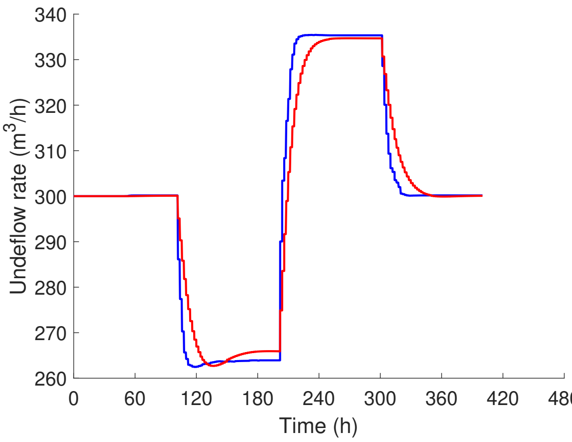
The ISE for the standard MPC and DDPC were 54.03 and 37.40, respectively. These metric values indicate that the DDPC managed to track the setpoint more effectively. Furthermore, to analyze the controller's control effort to achieve the good tracking performance, we calculated their respective integral of the absolute derivative of the control signal (IADU), given by

$$IADU = \int_0^\infty \left| \frac{du(t)}{dt} \right| dt. \tag{29}$$

The IADU for the standard MPC and DDPC were 141.91 and 139.55, respectively. The metric values were quite similar. This performance comparison between the controllers indicates that a DDPC based on samples dataset can yield a good control performance as good as a standard MPC using the process model. The advantage of the data-driven predictive control, in this work represented by the LACKI predictive controller, relies on measurement samples possible to be acquired during the process operation, while the standard MPC relies on a model that might not be available to such a process as the thickening process.



(a) .



(b)

Figure 4. Evolution of the (a) controlled variable and (b) manipulated variables during setpoint tracking (black dash line) for the DDPC (blue line) and conventional MPC (red line).

5. CONCLUSION

This work presented a data-driven predictive control approach, which integrates the formulation of MPC and the prediction technique LACKI. This approach is shown to be a viable alternative for controlling processes whose dynamics models are not reliable, in this work, represented by a thickening process. Comparative experiments were carried out with a standard MPC and the DDMPC. While the former used a linearized model of the process to control it, the latter used a dataset built by measured samples during the process operations. Simulated results indicated similar performance between the controllers, both managing to maintain the controlled variable close to the setpoint. It indicated that in case of the lack of an accurate model of a thickening process, which is most likely to be the case, there is still a way to utilize MPC to control such a process by using data-driven predictive control. For future works, we intend to test the LACKI predictive control in an MIMO system control by adding the interface level control.

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